Induction:

Inductive Hypothesis (I.H.) Assume that P(k) is true for any k, where k ≥ 3. It is required to prove, P(k+1), which is for any k where k ≥ 3, 2(k+1) + 1< 2k+1

According to the I.H.,

For any k where k ≥ 3, (2k+1) < 2k (\*)

For any k where k ≥ 3, 21 < 2k (\*\*)

Adding (\*) with (\*\*),

For any k where k ≥ 3, (2k+1) + 21 < 2k + 2k

Or, (2k+1) + 2 < 2×2k

Or, 2(k+1) + 1 < 21×2k

According to the Rule of Index, for the same base, the powers add up in multiplication. Therefore, applying the Rule of Index:

2(k+1) + 1 < 21+k

Or, 2(k+1) + 1 < 2k+1

Hence, P(k+1) is also true.

Q.E.D.